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Effect of Solar Angles on Incident Energy of the Flat Collectors

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Abstract

In this paper the daily, monthly, seasonally, and yearly optimum slope angles of solar collectors are determined for areas in Iran and new models are developed to calculate the monthly, seasonally, and yearly optimum slope angles for latitudes of $20°$ to $40°$ north. To achieve this purpose, the slope and surface azimuth angles of solar collectors for receiving maximum solar radiation were determined in some Iranian cities in different days, months, seasons, and the whole year employing different models. According to the optimum slope angles predicted in this paper and using the optimum slope angles achieved by other researchers at locations out of Iran but in the same range of latitudes, the models are obtained. The outcome of this research is that the optimum slope angle of flat solar collectors has a linear relationship with the latitude of the site.

Keywords: Maximum solar energy, Optimum slope angle, Azimuth angle, Solar collector, New models.

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1. Introduction

To receive maximum solar energy, the collector's surface should be perpendicular to the sun's rays. This can be accomplished when the solar trackers are used to follow the sun instantaneously. The main problem in this regard is however the high cost of this kind of trackers; so instead of employing solar trackers, the angles of collector's surface could be changed manually every day or month or season in order to adjust the collector almost perpendicular to the sun's rays. The majority of studies in this field focused on the monthly slope angle of the solar collectors and the results show that the slope angle depends on latitude. As an example, Heywood [1] obtained the yearly optimum angle as $\beta_{opt(y)} = \phi - 10$, Lunde [2] achieved this angle as $\beta_{opt,y} = \phi \pm 15$ and Duffie and Beckman [3] calculated this angle as $\beta_{opt(y)} = (\phi + 15) \pm 15$. Qiu and Riffat [4] found the yearly optimum tilt angle of solar collectors as $\beta_{opt(y)} = \phi \pm 10$ at a location with latitude of ϕ and the solar energy gain calculated based on the above angles had a relative error below 1.5%. In the above equations, the plus or minus sign depends on the season of the year and the hemisphere of the earth. For example, in places located at the north hemisphere the plus sign should be used in winter. Nijegorodov et al. [5] presented 12 equations for calculating the monthly optimum slope angle which is used in subsequent studies for validation of other researchers' results. He used the atmospheric transmittance models to obtain analytical formulas for the optimum slope angle. On the other hand, he used mathematical models for calculating the hourly total radiation and then integrated them to obtain total daily radiation. The atmospheric transmittance models may not be accurate for all climates. He also used some simplifying assumptions for employing the equations. The equations are therefore not too accurate and have a big deviation from the exact values in some latitudes and especially in some months.

Morcos [6] obtained a mathematical model for calculating the total radiation on a sloped surface and determined the optimum tilt angles for a flat plate collector in Assiut, Egypt on a daily basis. The results showed that changing the tilt angle eight times a year is necessary to receive the total radiation on the collector near its maximum value and this achieved a yearly gain of 6.85% in total radiation when compared with a flat plate collector fixed at slope of 27°, which is equal to the latitude of Assiut. Abdulaziz [7] computed the optimum slope angle for latitudes ranging from 10° to 50° north and concluded that if the collector is adjusted at the yearly optimum slope angle, the energy gain is less than 10% as compared to the monthly optimum slope angle. Furthermore, the optimum seasonal slope angle reduces the energy gain by less than 2% from that of the monthly optimum slope angle. Hartley et al. [8] calculated the optimum slope angle for Valencia, Spain. They showed that the amount of irradiation loss using the yearly average optimum tilt angle is only 6% when compared with the monthly average tilt angle, and thus using the yearly optimum angle may be preferred because it would need cheaper equipment and involve less work to keep the tilt angle the same all year round. Oladiran [9] determined the average global radiation on flat surfaces for three zones in Nigeria. The total radiation was obtained while the surface azimuth angle was varied between 0° and 75° at 15° intervals. He also presented the results for three slope angles of the collector surface and found that the mean annual radiation increased for a surface with the slope angle of 10° less than the latitude angle. Azmi et al. [10] computed the monthly optimum slope angle for Brunei, Darussalam. Their results had significant difference with Nijegorodov equations in some months because the ambient condition of Ref. [10] is not the same as assumptions of Nijegorodov. Shariah et al. [11], by employing the computer program TRNSYS (Transient System Simulation) found the optimum slope angle for a thermosyphon solar water heater installed in northern and southern parts of Jordan. Runsheng Tang et al. [12] presented an estimation of the optimal tilt angle for maximizing its energy based on the monthly global and diffuse radiation on a horizontal surface. They employed a mathematical model for the estimation of the optimal tilt angle of a collector and presented a contour map of the optimal tilt angles of the south-facing collectors used for the whole year in China, based on monthly horizontal radiation of 152 places around the country. Ulgen [13] computed the monthly, seasonally, and yearly optimum slope angles for Izmir, Turkey using a mathematical model. He found that the optimum tilt angle changes between 0 (June) and 61 degrees (December) throughout the year. Elminir et al. [14] studied the optimum slope angle theoretically in Helwan, Egypt and compared the results of different mathematical models with experimental results. The predictions have a little deviation from the experimental results. Gopinathan et al. [15] presented the monthly average daily global radiation on surfaces tilted towards the equator and also inclined at various azimuth angles for three locations in the South African region. They found that Maximum energy occurred at an azimuth angle of 180° (facing equator) at any slope, because these African cities are located at the southern hemisphere. Gunerhan and Hepbasli [16] calculated the daily optimum slope angle for Izmir, Turkey and compared the results with the results achieved from Nijegorodov equations, even though, the ambient condition in Izmir is different from Nijegorodov assumptions. They suggested that if we can change the slope angle once a month and adjust the collector at the monthly average slope angle, the utilization efficiency of solar collectors will increase. In 2009, Skeiker [17] obtained an equation for calculating the optimum daily slope angle and employed it to compute this angle for some cities in Syria. His results were validated when compared with Nijegorodov equations and as described before, the ambient condition in Syria is different from Nijegorodov assumptions. In 2011, Talebizadeh et al. [18] developed new models to predict optimum slope angle of solar collectors for different latitudes in Iran. They determined the optimum angles of 21 cities with the latitudes ranging from 25 to 38 for developing the correlations. Their results are more accurate than the previous models for this range of latitudes. In 2011, Talebizadeh et al. [19] employed the genetic algorithm to predict the optimum slope angle of solar collectors and photovoltaic panels. They also determined the hourly, daily, monthly and yearly optimum slope angles and determined the energy gain of each case.

The aim of this paper is to develop a new model for each month and to calculate the optimum monthly slope angle that is more accurate than Nijegorodov equations for latitudes of 20° to 40° north. For this purpose, the data of the optimum angles of different locations in this range of latitudes are needed. So, six Iranian cities, Kerman, Yazd, Zahedan, Birjand, Shiraz, and Tabas, were selected to find their optimum angles (5 cities for obtaining the models and 1 for validating). This information and the data of 5 other locations in the above range of latitudes gathered from the literature were used to develop the models. The models were developed to predict the seasonally and yearly optimum slope angles.

2. Mathematical modelling

The data of radiative energy on the horizontal surface are usually available and can be applied to calculate the energy received on an inclined surface. The total monthly average daily radiation $\overline{H}_T$ is the sum of direct, diffuse, and reflecting components according to [3].

$$\overline{H}_T = \overline{H}_b + \overline{H}_d + \overline{H}_r \tag{1}$$

The first model applied in this paper for calculating $\overline{H}_T$ is that of Liu and Jordan [20] as extended by Klein [21], which has been widely used and reputed as isotropic method. In this model, the diffuse and ground-reflected radiations are each assumed isotropic and monthly average daily radiation $\overline{H}_T$ is introduced as follows:

$$\overline{H}_T = \overline{H}\left(1-\frac{\overline{H}_d}{\overline{H}}\right)\overline{R}_b + \overline{H}_d\left(\frac{1+\cos\beta}{2}\right) + \overline{H}\rho_g\left(\frac{1-\cos\beta}{2}\right) \tag{2}$$

The major angles of solar collectors are shown in Fig. 1.

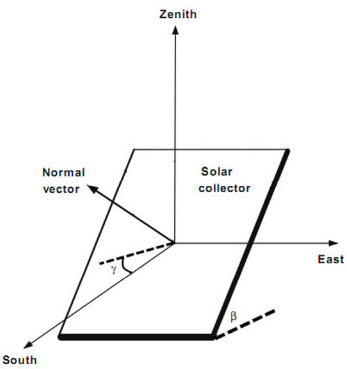

Fig. 1. The major angles of solar collectors

The ratio of the average daily direct radiation on an inclined surface to that on a horizontal surface for the month is $\overline{R}_b$ which is determined according to:

$$\overline{R}_b = \frac{\overline{H}_{bT}}{\overline{H}_b} = \frac{cos(\phi-\beta)cos\delta\, sin\omega'_s + \frac{\pi}{180}\omega'_s\, sin(\phi-\beta)sin\delta}{cos\phi\, cos\delta\, sin\omega_s + \frac{\pi}{180}\omega_s\, sin\phi\, sin\delta} \tag{3}$$

ω'_s in the above equation, is defined as follows:

$$\omega'_s = min\begin{bmatrix} cos^{-1}(-tan\phi\, tan\delta) \\ cos^{-1}(-tan(\phi-\beta)tan\delta) \end{bmatrix} \tag{4}$$

This model is used for collectors with $\gamma = 0$. An alternative model for calculating the monthly average daily radiation on an inclined surface has been developed by Klein and Thiacker (the KT model) [22]. This model in general considers both the slope and azimuth angles and according to Duffie and Beckman [3], it gives improved results, compared with the isotropic method. The total monthly average daily radiation $\overline{H}_T$ is defined as follows:

$$\overline{H}_T = \overline{H}\,\overline{R} \tag{5}$$

The equation for $\overline{R}$ is:

$$\overline{R} = D + \frac{\overline{H}_d}{\overline{H}}\left(\frac{1+\cos\beta}{2}\right) + \rho_g\left(\frac{1-\cos\beta}{2}\right) \tag{6}$$

Where:

$$D = \begin{Bmatrix} \max\{0, G(\omega_{ss},\omega_{sr})\} if \omega_{ss} \geq \omega_{sr} \\ \max\{0, [G(\omega_{ss},-\omega_s) + G(\omega_s,\omega_{sr})]\} if\ \omega_{ss} \leq \omega_{sr} \end{Bmatrix} \tag{7}$$

The three values of G applied in the above equation are defined as:

$$G(\omega_1,\omega_2) = \frac{1}{2d}\begin{bmatrix} \left(\frac{bA}{2} - a'B\right)(\omega_1-\omega_2)\frac{\pi}{180} + \\ (a'A - bB)(sin\omega_1 - sin\omega_2) - \\ a'C(cos\omega_1 - cos\omega_2) - \\ \left(\frac{bA}{2}\right)(sin\omega_1 cos\omega_1 - sin\omega_2 cos\omega_2) \\ + \frac{bC}{2}(sin^2\omega_1 - sin^2\omega_2) \end{bmatrix} \tag{8}$$

Sunrise angle ω_{sr} and sunset angle ω_{ss} are introduced as:

$$|\omega_{sr}| = min\left[\omega_s, cos^{-1}\frac{AB + C\sqrt{A^2 - B^2 + C^2}}{A^2 + C^2}\right] \tag{9}$$

$$\omega_{sr} = \begin{Bmatrix} -|\omega_{sr}|\ if(A>0\ and\ B>0)\ or\ (A \geq B) \\ +|\omega_{sr}|\ otherwise \end{Bmatrix} \quad (9\text{-a})$$

$$|\omega_{ss}| = min\left[\omega_s, cos^{-1}\frac{AB - C\sqrt{A^2 - B^2 + C^2}}{A^2 + C^2}\right] \quad (10)$$

$$\omega_{ss} = \begin{Bmatrix} +|\omega_{ss}|\ if(A>0\ and\ B>0)\ or\ (A \geq B) \\ -|\omega_{ss}|\ otherwise \end{Bmatrix} \quad (11)$$

Where:

$$A = cos\,\beta + tan\,\phi\, cos\,\gamma\, sin\,\beta \quad (12)$$

$$B = cos\,\omega_s\, cos\,\beta + tan\,\delta\, sin\,\beta\, cos\,\gamma \quad (13)$$

$$C = \frac{sin\,\beta\, sin\,\gamma}{cos\phi} \quad (14)$$

And

$$\omega_s = \cos^{-1}(-\tan\phi\tan\delta) \quad (15)$$

Parameters a, a', b, d are defined as follows:

$$a = 0.4090 + 0.5016\, sin(\omega_s - 60) \quad (16)$$

$$b = 0.6609 - 0.4767\sin(\omega_s - 60) \quad (17)$$

$$a' = a - \frac{\overline{H}_d}{\overline{H}} \quad (18)$$

$$d = \sin(\omega_s) - \frac{\pi\omega_s}{180}\cos(\omega_s) \quad (19)$$

It is worth mentioning that the above equations for calculating the monthly average daily radiation, $\overline{H}_T$ can also be applied for total daily radiation, H_T on an inclined surface provided that the parameters used in each equation are defined on a daily basis. In the above models, the monthly average clearness index $\overline{K}_T$ is applied for calculating $\overline{H}_d$ which is defined as the ratio of monthly average daily radiation on a horizontal surface to the monthly average daily extraterrestrial radiation.

$$\overline{K}_T = \frac{\overline{H}}{\overline{H}_0} \quad (20)$$

$\overline{H}_d$ is calculated as:

$$for\ \omega_s \leq 81.4°\ and\ 0.3 \leq \overline{K} \leq 0.8$$
$$\frac{\overline{H}_d}{\overline{H}} = 1.391 - 3.56\overline{K} + 4.189\overline{K}^2 - 2.137\overline{K}^3 \quad (21)$$

$$for\ \omega_s > 81.4°\ and\ 0.3 \leq \overline{K} \leq 0.8$$
$$\frac{\overline{H}_d}{\overline{H}} = 1.311 - 3.022\overline{K} + 3.427\overline{K}^2 - 1.821\overline{K}^3 \quad (22)$$

$\overline{H}_0$ for latitudes of $+60°$ to $-60°$ can be calculated for the average day of the month as:

$$\overline{H}_0 = \frac{24 \times 3600 G_{sc}}{\pi}(1 + 0.033cos\frac{360n}{365}) \times (cos\phi\, cos\delta\, sin\omega_s + \frac{\pi\omega_s}{180} sin\phi\, sin\delta) \quad (23)$$

The daily clearness index K_T is defined as:

$$K_T = \frac{H}{H_0} \quad (24)$$

Where H_0 can be calculated using Eq. 23 when n and δ depend on the day of the month and H_d is calculated as [3]:

$$for\ \omega_s \geq 81.4°$$
$$\frac{H_d}{H} = 1 + 0.2832\,K_T - 2.5557\,K_T^{\ 2} + 0.8448\,K_T^{\ 3} \quad for\, K_T < 0.722$$
$$\frac{H_d}{H} = 0.175 \quad for\ K_T \geq 0.722 \quad (25)$$

$$for\ \omega_s \leq 81.4°$$
$$\frac{H_d}{H} = 1 - 0.2727\,K_T - 2.4495K_T^{\ 2} + 11.9514K_T^{\ 3} + 9.3879K_T^{\ 4} \quad for K_T < 0.722$$
$$\frac{H_d}{H} = 0.175 \quad for\ K_T \geq 0.722 \quad (26)$$

3. Results and discussion

New models are developed to predict the optimum slope angles for Iranian cities. The optimum slope angles obtained using these models are compared with the Nijegorodov results.

For calculating radiation components on an inclined surface, these data should be available on a horizontal surface first. This information is borrowed from the Iranian Meteorology Organization (IMO) in six Iranian cities for a period of 23 years from 1983 to 2005. These cities are Kerman, Yazd, Zahedan, Birjand, Shiraz, and Tabas and their latitudes are 30.15°, 31.54°, 29.28°, 32.52°, 29.32°, and 33.36° respectively. For receiving maximum solar radiation, the solar collector should be mounted at the optimum angles recommended according to the present calculations.

The mean daily solar radiation is calculated for each month and applied to determine the monthly optimum angles which are listed in Table 1.

3.1. Calculating the optimum angles employing two different models

As discussed in Section 2, the isotropic model (Klein) for monthly and daily radiation is used when the azimuth 1angle is zero. So, the optimum slope angle is only predicted in this model. Most of the relevant studies of did not consider the azimuth angle since it is assumed equal to zero for northern hemisphere according to Duffie and Beckman [3]. The KT model which calculates the amount of energy received on the collector considering both slope and azimuth angles is used here to find the optimum azimuth angle in addition to the optimum slope angle. The azimuth angle is considered here to control the calculations.

The optimum azimuth angle at different days and months of a year in Yazd and Birjand are calculated and shown in Figs. 2 and 3, respectively. It is worth mentioning that because of page limitation only the figures regarding γ_{opt} for Yazd and Birjand are shown. Figs. 2 and 3 show that the azimuth angle is almost equal to zero.

In this section, the optimum slope angle using two different models are investigated. These angles are achieved when applying the isotropic (KT) model without considering the azimuth angle, and the KT model with different azimuth angles.

Table 1. Monthly average daily solar radiation on a horizontal surface ($\overline{H}(MJ.m^{-2})$) for 6 Iranian cities

	Kerman	Yazd	Zahedan	Birjand	Shiraz	Tabas
Jan	12.52	13.07	13.38	13.59	13.92	11.56
Feb	15.83	17.33	16.33	15.83	15.64	13.84
Mar	18.36	18.96	18.93	18.84	18.00	16.80
Apr	23.00	23.24	24.27	22.00	22.21	21.40
May	26.83	26.95	26.15	26.36	25.24	23.86
Jun	28.54	29.73	27.18	27.56	27.72	26.11
Jul	28.10	28.79	27.97	28.13	25.91	25.83
Aug	25.90	27.04	25.93	26.74	24.83	24.18
Sep	23.58	24.56	23.99	23.74	22.82	21.50
Oct	19.32	20.20	19.20	17.97	18.56	17.49
Nov	15.20	14.07	15.12	14.05	13.95	12.84
Dec	13.19	11.91	12.68	12.99	13.32	10.84

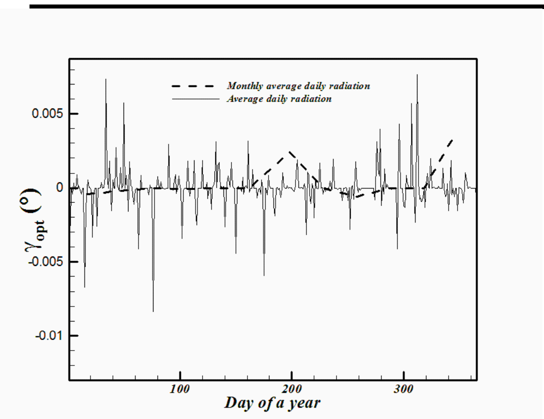

Fig. 2. The optimum azimuth angle at different days of a year for Yazd based on the mean optimum angle during the 22 year period for which the IMO data were available

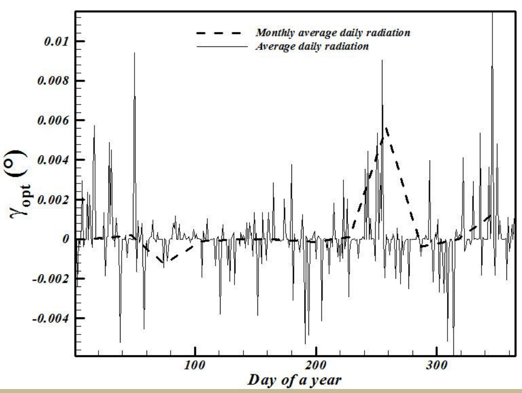

Fig. 3. The optimum azimuth angle at different days of a year for Birjand based on the mean optimum angle during the 22 year period for which the IMO data were available

Figs. 4 and 5 illustrate the values of $\beta_{opt(m)}$ and $\beta_{opt(d)}$ for Kerman, respectively and Figs. 6 and 7 show the values of $\beta_{opt(m)}$ and $\beta_{opt(d)}$ for Shiraz, respectively. As mentioned before, because of page limitation only the figures regarding the optimum slope angles for Kerman and Shiraz are shown.

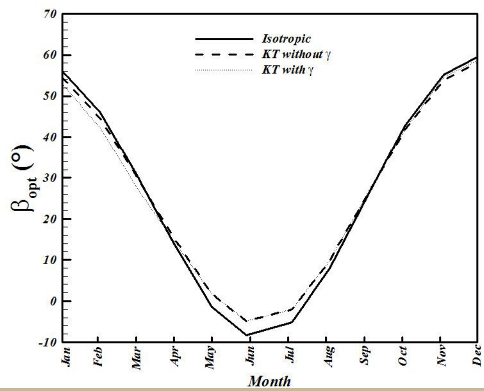

Fig. 4. The mean optimum slope angle at different months of a year employing different models for Kerman

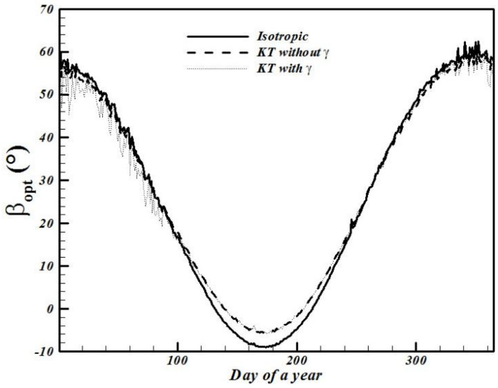

Fig. 5. The mean optimum slope angle at different days of a year with employing models for Kerman

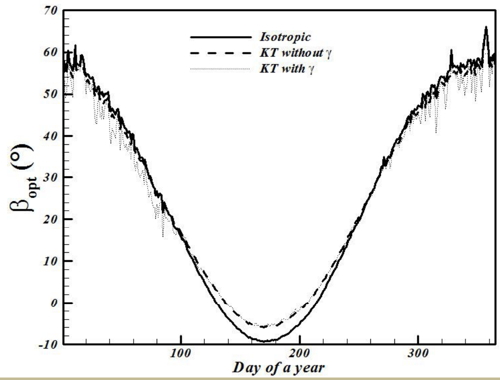

Fig. 7. The mean optimum slope angle at different days of a year employing different models for Shiraz

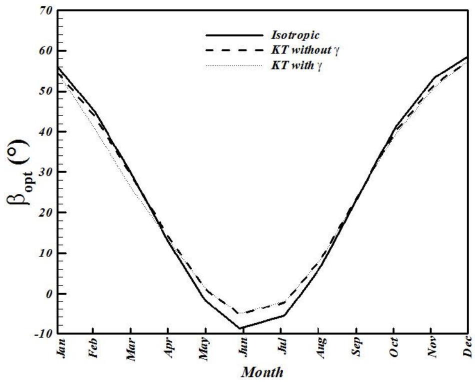

Fig. 6. The mean optimum slope angle at different months of a year employing different models for Shiraz

The results obtained from the KT model show that the optimum slope angle with or without considering the azimuth angle are close to each other and this confirms the accuracy of the calculations. The results of optimum slope angle at different months of a year for receiving maximum solar energy using the KT model are listed in Table 2 for all the above cities. Note that when the sign of the angle varies from positive to negative, it means that the collector surface direction is changed from the north to the south.

The optimum slope angle for the maximum solar energy is different for different months and also for different cities. The results of $\beta_{opt(s)}$ and $\beta_{opt(y)}$ are listed in Table 3.

Table 2. The values of slope angles at different months of a year with KT model

	Zahedan	Birjand	Shiraz	Tabas	Yazd	Kerman
Jan	54.14	58.37	54.64	57.69	56.72	52.83
Feb	44.00	47.60	40.48	47.82	47.59	42.31
Mar	30.01	33.28	26.22	33.07	32.50	27.83
Apr	14.71	17.25	13.34	17.87	16.65	14.55
May	0.97	3.89	1.31	4.68	2.98	1.77
Jun	-5.28	-2.80	-5.23	-1.94	-3.91	-4.89
Jul	-2.74	-0.10	-2.07	0.88	-0.97	-2.08
Aug	9.02	12.24	8.79	12.65	11.32	9.83
Sep	25.53	28.92	24.96	28.80	28.21	26.63
Oct	40.64	43.66	39.57	44.32	44.04	41.76
Nov	52.75	55.92	51.01	55.97	54.72	54.67
Dec	56.62	60.94	57.50	60.15	58.80	58.62

Table 3. The values of $\beta_{opt(s)}$ and $\beta_{opt(y)}$ calculated by the KT model

	Zahedan	Birjand	Shiraz	Tabas	Yazd	Kerman
The first quarter	42.72	46.42	40.45	46.19	45.60	40.99
The second quarter	3.46	6.11	3.14	6.87	5.24	3.81
The third quarter	10.60	13.69	10.56	14.11	12.85	-0.69
The forth quarter	50.00	53.51	49.36	53.48	52.52	51.68
Whole year	26.70	29.93	25.88	30.16	29.05	23.95

The values of $\beta_{opt(s)}$ are achieved by averaging the monthly optimum slope angles for each quarter and the value of $\beta_{opt(y)}$ is achieved by averaging the monthly optimum slope angles in the whole year. It is notable that the first quarter consists of January, February, and March, the second quarter consists of April, May, and June, the third quarter consists of July, August, and September, and the forth quarter consists of October, November, and December.

As mentioned before, if the collector is perpendicular to the sun's rays, it can receive more energy from the sun. So, it is expected that collector A (mounted at daily optimum slope angle) receives more energy than collector B, collector B (mounted at monthly optimum slope angle) receives more energy than collector C, and collector C (mounted at seasonally optimum slope angle) receives more energy than collector D (mounted at yearly optimum slope angle). Fig. 8 displays the energy received by a horizontal collector and collector D, and Table 4 gives the percentage of energy gain for collectors A, B, and C in comparison with collector D in Kerman, Yazd, Tabas, Shiraz, Birjand, and Zahedan, respectively.

Table 4. The energy gain on an inclined surface compared with collector D in cities of Kerman, Yazd, Tabas, Shiraz, Birjand, and Zahedan in a year

	Seasonal (C) (%)	Monthly (B) (%)	Daily (A) (%)
Kerman	2.84	3.66	4.05
Yazd	5.13	6.25	6.89
Tabas	4.76	5.77	6.15
Shiraz	4.97	6.01	6.89
Birjand	5.16	7.55	8.20
Zahedan	4.94	6.01	6.54

As expected, collector A receives the most energy from the sun. However, the energy received by collector A is only 1% higher than the energy received by collector B. It is evident that changing the slope angles in a daily manner by applying solar trackers is not economical, because of the high cost of trackers and low heat gain percentage.

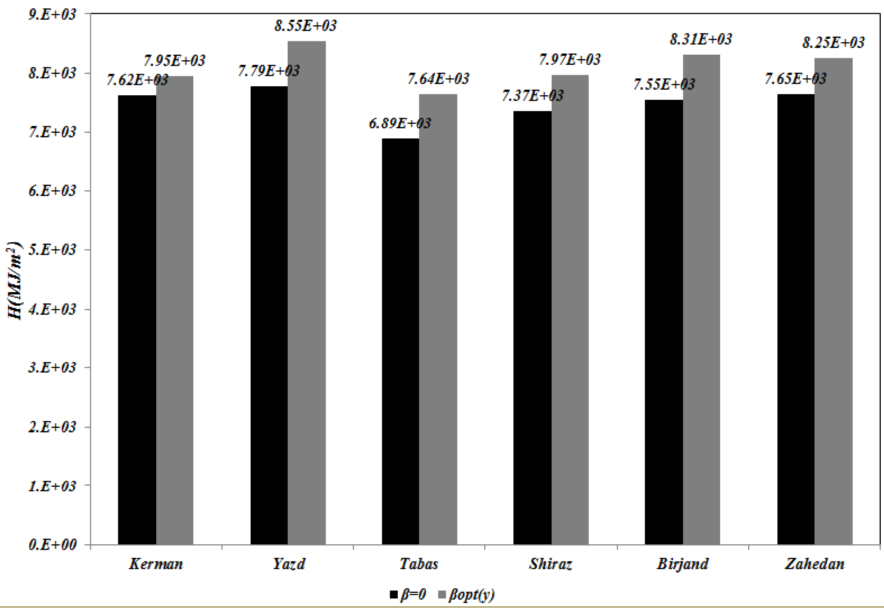

Fig. 8. The energy received on a horizontal surface and on collector D in a year

Note that changing the angles in order to gain more energy is more pronounced when a large number of collectors are involved.

In Figs. 9, 10, and 11, the energy received by the collector is displayed with respect to γ at four different slope angles $\phi-10°, \phi, \phi+10°, \phi+20°$ employing the KT model for Kerman, Tabas, and Zahedan, respectively. Considering Fig. 3, the maximum deviation from zero for γ is related to September, so Figs. 9, 10, and 11 are shown for this month.

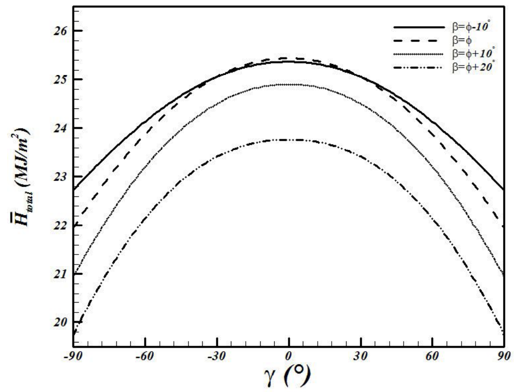

Fig. 9. The monthly mean daily radiation versus γ at different slope angles for Kerman in September

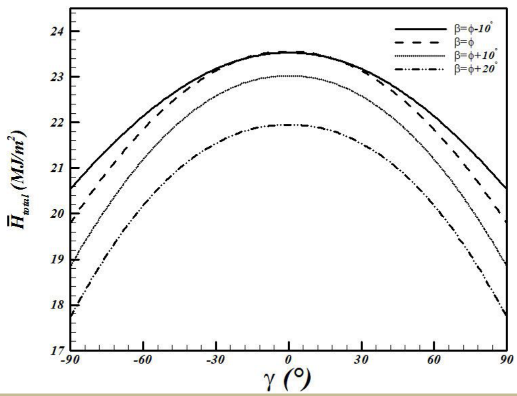

Fig. 10. The monthly mean daily radiation versus γ at different slope angles for Tabas in September

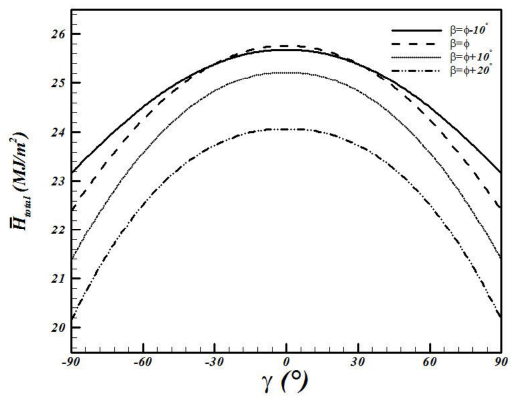

Fig. 11. The monthly mean daily radiation versus γ at different slope angles for Zahedan in September

It is observed from Figs. 9, 10, and 11 that the maximum received energy occurs in $\gamma=0$ and $\beta=\phi-10$ for all cities in September. In addition, by increasing the azimuth angle in positive or negative direction, the energy is decreased and by increasing the slope angle it decreases too. It is worth mentioning that the optimum slope angle and its effect on the received energy are different for different months. For example, in Fig. 12, the energy for different values of β is illustrated with respect to γ for Kerman in January. As shown in Fig. 12, the optimum slope angle is $\phi+20$ and by decreasing β, the received energy is decreased too.

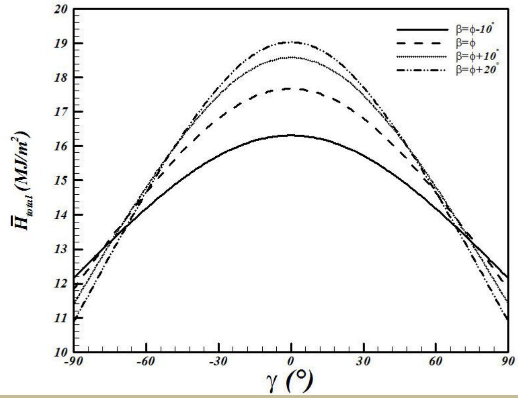

Fig. 12. The monthly mean daily radiation versus γ at different slope angles for Kerman in January

3.2. Developing models to calculate the monthly optimum slope angle

As mentioned before, Nijegorodov et al. [5] gave a set of 12 equations to calculate the monthly optimum slope angles for any location having the latitudes of 60° south to 60° north. In this paper, 12 equations were obtained for the calculation of monthly optimum slope angle for the latitudes of 20° to 40° north which are more accurate than Nijegorodov's equations. The optimum slope angles for five cities considered in this paper and five foreign cities, their latitudes in the above range, are used to develop these models. The foreign cities are Dhaka, Bangladesh [23], Assiut, Egypt [6], Damascus, Syria [17], Izmir, Turkey [16], and Valencia, Spain [8] with the latitudes of 23.73°, 27°, 33.40°, 38.46°, and 39.50° respectively.

The correlations were obtained by fitting the best curve matching these data points. For example, the data and the fitted line in January and October are illustrated in Figs. 13 and 14, respectively.

The models for calculating $\beta_{opt(m)}$ as a function of ϕ are listed in Table 5. It is worth mentioning that $\beta_{opt(m)}$ is a function of latitude as recommended by Duffie and Beckman [3] and Nijegorodov [5], because γ_{opt} is equal to zero and some parameters such as solar radiation on horizontal surface depends on the latitude.

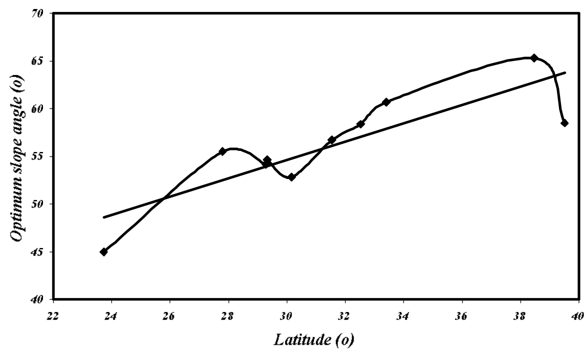

Fig. 13. Curve fitting procedure to obtain the linear correlation in January

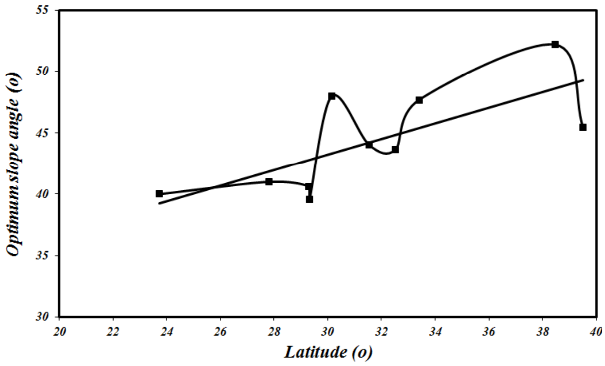

Fig. 14. Curve fitting procedure to obtain the linear correlation in October

Table 5. The model equations for monthly optimum slope angles

Month	Present equations
January	$\beta_{opt(m)} = 0.9901\phi + 24.631$
February	$\beta_{opt(m)} = 0.6613\phi + 26.283$
March	$\beta_{opt(m)} = 1.2657\phi - 8.6368$
April	$\beta_{opt(m)} = 0.89\phi - 11.878$
May	$\beta_{opt(m)} = 0.381\phi - 9.3689$
June	$\beta_{opt(m)} = 0.0235\phi - 2.9196$
July	$\beta_{opt(m)} = 0.138\phi - 4.2233$
August	$\beta_{opt(m)} = 0.3931\phi - 0.4064$
September	$\beta_{opt(m)} = 0.1767\phi + 23.08$
October	$\beta_{opt(m)} = 0.6592\phi + 23.08$
November	$\beta_{opt(m)} = 0.9975\phi + 23.192$
December	$\beta_{opt(m)} = 0.9236\phi + 29.184$

Some collectors should be fixed and their slope angle cannot be changed each month, and changing each season is adequate to receive the maximum energy. So, calculating the optimum seasonal and yearly slope angles is also important. The correlations for computing $\beta_{opt(s)}$ and $\beta_{opt(y)}$ are given in Table 6.

Table 6. The model equations for seasonally and yearly optimum slope angles

The first quarter	$\beta_{opt(s)} = 1.073\phi + 10.3$
The second quarter	$\beta_{opt(s)} = 0.4885\phi - 10.27$
The third quarter	$\beta_{opt(s)} = 0.2631\phi + 4.961$
The forth quarter	$\beta_{opt(s)} = 0.8966\phi + 23.81$
Year	$\beta_{opt(y)} = 0.6804\phi + 7.203$

3.3. Comparison of the present results with the exact values and the reference [5] results

As mentioned in the Introduction Section, Nijegorodov used mathematical models for calculating the hourly total radiation and then integrated them to obtain the total daily radiation. Firstly, the atmospheric transmittance models he applied in his paper may not be accurate for all climates and locations. Secondly, he assumed that $\overline{K_T}$ is fixed and equal to 0.7. This assumption however causes some discrepancies, because the clearness index is not fixed and it varies between 0.5 and 0.7 for the Iranian cities under consideration. In this section the optimum slope angles achieved experimentally at Cairo, Egypt [24] with the latitude of 29.52° is used to verify the accuracy of present correlations first, and then these results are compared with Nijegorodov results to show that the present results are more accurate than his. It is worth mentioning that the results of Cairo, Egypt and Tabas, Iran were not used in model development.

Table 7 gives the experimental results of monthly optimum slope angle for Cairo, Egypt [24], the Nijegorodov results and the present results at the same latitude and also the deviation of the results at each month. As given in Table 7, the present results are in a close agreement with the experimental results and they are also more accurate than the reference [5] results. Table 8 gives the present results for Tabas, Iran which are more accurate than the reference [5] results.

Finally, the results for Zahedan, Iran and Valencia, Spain that were used in model developments are given in Tables 9 and 10, respectively. The figures are also subjective to higher accuracy of the present results. However, the achieved equations might not be more accurate in certain month. It is notable that the last rows of Tables 7-10 indicate the RMSE to show the calculated quantity of the errors for each city.

Table 7. The comparison of present slope angles with the exact results and reference [5] for Cairo

Cairo, Egypt	Experimental Results	Nijegorodov	Deviation of Nijegorodov results	Present models	Deviation of present results
Jan	51.00	55.27	-4.27	53.86	-2.86
Feb	48.00	45.63	2.37	45.80	2.20
Mar	33.00	33.52	-0.52	28.73	4.27
Apr	21.00	19.52	1.48	14.39	6.61
May	4.00	3.45	0.55	1.88	2.12
Jun	4.00	-8.32	12.32	-2.23	6.23
Jul	7.00	-3.73	10.73	-0.15	7.15
Aug	20.00	11.63	8.37	12.01	7.99
Sep	32.00	27.52	4.48	28.51	3.49
Oct	48.00	41.52	6.48	42.54	5.46
Nov	53.00	52.45	0.55	52.64	0.36
Dec	55.00	59.68	-4.68	56.45	-1.45
RMSE		6.11		4.81	

Table 8. The comparison of present slope angles with the exact results and reference [5] for Tabas

Tabas, Iran	Calculated results	Nijegorodov	Deviation of Nijegorodov results	Present models	Deviation of present results
Jan	57.69	58.69	-1.00	57.66	0.03
Feb	47.82	49.36	-1.54	48.34	-0.52
Mar	33.07	37.36	-4.29	33.59	-0.52
Apr	17.87	23.36	-5.49	17.81	0.06
May	4.68	7.02	-2.34	3.34	1.34
Jun	-1.94	-4.98	3.04	-2.14	0.20
Jul	0.88	-0.31	1.19	0.38	0.50
Aug	12.65	15.36	-2.71	13.52	-0.87
Sep	28.80	31.36	-2.56	29.19	-0.39
Oct	44.32	45.36	-1.04	45.07	-0.75
Nov	55.97	56.02	-0.05	56.47	-0.50
Oct	60.15	63.02	-2.87	60.00	0.15
RMSE		2.76		0.6	

Table 9. The comparison of present slope angles with the exact results and reference [5] for Zahedan

Zahedan, Iran	Calculated results	Nijegorodov	Deviation of Nijegorodov results	Present models	Deviation of present results
Jan	54.14	55.06	-0.92	53.62	0.52
Feb	44.00	45.40	-1.40	45.65	-1.65
Mar	30.01	33.28	-3.27	28.42	1.59
Apr	14.71	19.28	-4.57	14.18	0.53
May	0.97	3.23	-2.26	1.79	-0.82
Jun	-5.28	-8.53	3.25	-2.23	-3.05
Jul	-2.74	-3.94	1.20	-0.18	-2.56
Aug	9.02	11.40	-2.38	11.92	-2.90
Sep	25.53	27.28	-1.75	28.47	-2.94
Oct	40.64	41.28	-0.64	42.38	-1.74
Nov	52.75	52.23	0.52	52.40	0.35
Dec	56.62	59.47	-2.85	56.23	0.39
RMSE		2.4		1.9	

Table 10. The comparison of present slope angles with the exact results and reference [5] for Valencia

Valencia, Spain	Calculated results	Nijegorodov	Deviation of Nijegorodov	Present models	Deviation of present results
Jan	58.5	64.16	-5.66	63.74	-5.24
Feb	49	55.32	-6.32	52.40	-3.40
Mar	35	43.50	-8.50	41.36	-6.36
Apr	20	29.50	-9.50	23.28	-3.28
May	4.5	12.74	-8.24	5.68	-1.18
Jun	0	0.37	-0.37	-1.99	1.99
Jul	3	5.16	-2.16	1.23	1.77
Aug	14.5	21.32	-6.82	15.93	-1.43
Sep	29	37.50	-8.50	30.28	-1.28
Oct	45.5	51.50	-6.00	49.12	-3.62
Nov	60	61.74	-1.74	62.59	-2.59
Dec	61	68.37	-7.37	65.67	-4.67
RMSE		6.58		3.46	

4. Conclusions

In this paper, 12 equations were developed for calculating monthly optimum slope angles at latitudes of 20° to 40° north which are more accurate than the previous models published in the literature. To obtain the correlations, the monthly optimum slope angles of five Iranian cities are first computed numerically using the meteorological data [25]. These results and the values of monthly optimum slope angles of five foreign cities with almost the same latitudes were used. Curve fitting procedure was used to find a linear model to approximate the data points. The present results were more accurate than the reference data, because of some simplifying assumptions used in reference [5] models. The following remarks were concluded in the course of this study:

The optimum slope angle of flat solar collectors has a linear relationship with the latitude of the site.

The maximum solar energy in different days of a year happens at different slope angles.

The calculations showed that more energy is gained on an inclined collector by mounting the collector at optimum daily angle than monthly, seasonally, and yearly angles. However, the energy gains using the daily and monthly optimum angles are almost the same. So the slope angle can be changed each month in order to gain more energy than a horizontal surface or even a surface with yearly optimum angle.

Comparison of two different methods showed the veracity of the present results. The difference is due to some simplifying assumptions applied in isotropic model.

Nomenclature

$\overline{H}$ Monthly average daily radiation on a horizontal surface $(MJ.m^{-2})$

H Daily radiation on a horizontal surface $(MJ.m^{-2})$

$\overline{H}_T$ Total monthly average daily radiation on an inclined surface $(MJ.m^{-2})$

H_T Total daily radiation on an inclined surface $(MJ.m^{-2})$

$\overline{H}_b$ Monthly average daily beam radiation on a horizontal surface $(MJ.m^{-2})$

$\overline{H}_d$ Monthly average daily defuse radiation on a horizontal surface $(MJ.m^{-2})$

$\overline{H}_r$ Monthly average daily ground reflected radiation on a horizontal surface $(MJ.m^{-2})$

H_d Daily defuse radiation on a horizontal surface $(MJ.m^{-2})$

$\overline{H}_0$ Monthly daily extraterrestrial radiation on a horizontal surface $(MJ.m^{-2})$

H_0 Daily extraterrestrial radiation on a horizontal surface $(MJ.m^{-2})$

$\overline{K}_T$ The monthly average clearness index

K_T The daily average clearness index

n Day of a year

Greek symbol

β Slope angle (°)

$\beta_{opt(d)}$ Daily optimum slope angle (°)

$\beta_{opt(m)}$ Monthly optimum slope angle (°)

$\beta_{opt(s)}$ Seasonally optimum slope angle (°)

$\beta_{opt(y)}$ Yearly optimum slope angle (°)

γ Surface azimuth angle(°)

γ_{opt} Optimum surface azimuth angle (°)

ϕ Latitude (°)

δ Declination angle (°)

ω'_s Sunset hour angle (°)

ω_{sr} Sunrise angle (°)

ω_{ss} Sunset angle (°)

ρ_g Ground reflectance factor

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